Solutions

36. *Proposed by D.M. Bătineţu-Giurgiu, Matei Basarab National College, Bucharest, Romania and Neculai Stanciu, George Emil Palade General School, Buzău, Romania.* Prove that if p is a prime number ($p > 3$), then the number $p^2 + 2015$ is multiple of 24.

Solution by Arkady Alt, San Jose, California, USA. First note that $p^2 - 1$ divisible by 8 for any odd p. Indeed, for $p = 2k + 1$we have $p^2 - 1 = 4k(k+1)$where $k(k+1)$ is even for any integer k. Since $p > 3$ is prime then p isn't divisible by 3 and, therefore, p can be represented in the form $p = 3k \pm 1$ for some integer k. Hereoff, $p^2 - 1 = 9k^2 \pm 6k = 3k(3k \pm 2)$. Since 3 and 8 are relatively prime and $p^2 - 1$ divisible by 8 and by 3 then $p^2 - 1$ divisible by 24. Noting that 2016 is divisible by 24 as well we can conclude that $p^2 + 2015 = p^2 - 1 + 2016$ is divisible by 24.

Also solved by Adnan Ali, Student in A.E.C.S-4, Mumbai, India, Henry Ricardo, New York Math Circle, New York, USA, Corneliu Mănescu-Avram, Transportation High School, Ploieşti, Romania, Ioan Viorel Codreanu, Satulung, Maramures, Romania, Alexandru - Andrei Cioc and Daniel Văcaru, Piteşti, Romania and the proposer

37. *Proposed by Traşcă Iuliana, Olt, Romania* . Let $x, y, z > 0$. Prove that

$$\frac{2x+2y+4z}{4x+4y+3z} + \frac{2x+4y+2z}{4x+3y+4z} + \frac{4x+2y+2z}{3x+4y+4z} \geq \frac{24}{11}$$

Solution by Corneliu Mănescu–Avram, Transportation High School, Ploieşti, Romania (expanded by the editor). Simplify by 2 and sum 1 to each fraction al left. We get

$$\frac{5x+5y+5z}{4x+4y+3z} + \frac{5y+5z+5x}{4y+4z+3x} + \frac{5z+5x+5y}{4z+4x+3y} \geq \frac{12}{11} + 3$$

that is

$$5(x+y+z)\left(\frac{1}{4x+4y+3z} + \frac{1}{4y+4z+3x} + \frac{1}{4z+4x+3y}\right) \geq \frac{45}{11}$$

The AM–HM inequality yields

$$5(x+y+z)\left(\frac{1}{4x+4y+3z} + \frac{1}{4y+4z+3x} + \frac{1}{4z+4x+3y}\right) \geq$$
$$\geq 5(x+y+z)\frac{9}{4x+4y+3z+4y+4z+3x+4z+4x+3y} = \frac{45}{11}$$

and the proof is complete.

Also solved by Adnan Ali (two proofs), Student in A.E.C.S-4, Mumbai, India, Alexandru–Andrei Cioc and Daniel Văcaru (jointly), Lucía Ma Li (student) IES Isabel de España, Las Palmas de Gran Canaria, Spain and Ángel Plaza Universidad de Las Palmas de Gran Canaria, Spain (jointly), Henry Ricardo (two proofs), New York Math Circle, New York,